During the period in which we were so anxious to carry on our scientific pursuits, I worked out the factors of the third and fourth millions.[1] Madame Courcier has offered to print them if the Institut will contribute a thousand francs towards the expenses. This is only 500 francs for each million, although the second million alone has required a sacrifice of 1000 francs. I have a little repugnance at proposing this request; nevertheless I shall gain nothing except the uncomfortable labor of correcting the proofs, which probably no one after my death would ever attempt. On the other hand, I have seen in the reports of the past year that 1500 francs was given for the printing of botanical works, and yet that the mathematical section did not make any expenditure of a similar nature.

Madame Courcier has a good opinion of the sale of this work, since the English have bought several copies of the second million; if this hope is not maintained, she will probably increase her demand to such an extent that it would seem perhaps advisable to profit by the offer which she now makes.

I have the honor of being, with respect,

Monsieur,

V. T. H. & T. O. S,[2]
BURCKHARDT

Below there is a memorandum in Delambre's hand:

"Granted 1000 fr. 7 November 1814."

The letter reads so much like several that have come to my attention in recent years that I feel that it may contain some suggestions of value and some words of encouragement to those who have to meet the problem at the present time.

QUESTIONS AND DISCUSSIONS.

Edited by C. F. GUMMER, Queen's University, Kingston, Ont., Canada.

NEW QUESTION.

The following question was asked by a member of the Ohio Section of the Association, at the meeting of April 20, 1922:

47. Can anyone teaching the theory of investment furnish references for bond valuation formulas when the income rate is a varying function of the time?

DISCUSSIONS.

In the discussion concerning definitions, Professor Allen draws attention to the inexcusable carelessness of many writers on algebra in the terminology of complex numbers. It seems a reasonable demand that an author should use the terms "real," "imaginary" and "complex" in such a way that it may be definitely determined whether or not a given number belongs to one of these classes; and it is rather surprising to learn how few writers of texts have taken the trouble to do this consistently.

The vague and unsettled use of the word "imaginary" in former times can scarcely be blamed for the present situation. It is true that negative numbers

[1] That is, of his *Table des diviseurs pour tous le nombres des premier, deuxième et troisième millions, avec les nombres premiers qui s'y trouvent,* which was published in complete form, at Paris, in 1817.

[2] Votre toute honoré et tout obeissant serviteur.

were at one time called imaginary, and that infinity is sometimes apologetically placed in that category. No doubt also, in popular speech, a cash balance becomes imaginary when it attains the value zero. Still, in careful mathematical speech, a certain definite usage has become established, and departures from this are for the most part the result of negligence.

It is, for instance, generally understood that "imaginary" is equivalent to "unreal," and that "complex" includes the real. When anyone admits these things on one page, and on the next writes "complex" for "imaginary," he is simply following the bad tradition that mars so many older text-books, of *forgetting to allow for the extreme cases.*

It would be easy, but unnecessary, to name some books of this type in which we can never be certain that the positive number system is not going to include zero, and where the inequality $x > y$ is or is not consistent with $x = y$ according to the caprice of the author. In geometry too a great deal of confusion commonly prevails about the limiting cases. Of the many writers who entreat the reader to accept as a triangle one that has a zero angle, how many take the trouble to warn him of the necessity of re-stating some elementary theorems? For instance, a triangle may have two angles equal, but no two sides equal.

The reader will of course not take too literally the "definition" of i as one of the roots of $x^2 + 1 = 0$. How do we know that this equation has two roots, and not more (as in quaternion theory)? The answer is that we have already at this stage decided that a root i exists, and that it shall obey certain formal algebraic laws. It is a consequence of these assumptions that $x^2 + 1 = 0$ only when $x = \pm i$. It is therefore a matter of choice whether we take i to be a one- or a two valued number.[1]

It may be noted that Professor Allen's last suggestion is exactly the opposite of one that is sometimes followed. For example, Harkness and Morley[2] denote by $\sqrt[n]{a}$ *any* nth root of a, and by a^x the exponential function of $x \log a$, where $\log a$ is the *principal* logarithm and hence a^x is one-valued. This plan has one inconvenient consequence: it makes an odd root of a negative number imaginary. It agrees with the general practice of making e^x one-valued.

The discussion by Professors Cajori and Miller arises from Professor Miller's former article on the same subject, and requires no comment.

I. Definitions of Imaginary and Complex Numbers.[3]

By Edward S. Allen, Iowa State College.

In reading certain parts of about 60 texts on algebra—those of Chrystal, Serret, and Weber among them[4]—I have discovered with surprise that the

[1] For the synthetic treatment of complex numbers as pairs of real numbers, see L. E. Dickson, *Elementary Theory of Equations*, p. 21.

[2] *Introduction to Analytic Functions*, London, 1898, p. 24, 168.

[3] Read at the April meeting of the Ohio Section of the Association.

[4] The list includes also books by all members of that subcommittee of the National Committee on Mathematical Requirements which made the admirable report on "Terms and symbols in elementary mathematics." Dickson's *First Course in the Theory of Equations*, published since this discussion was submitted to the editors of the Monthly, conforms to the policy here recommended in its elegant, brief exposition of the complex number system.